

The FPA-PM Objective Function and Escaped Fires:

A Briefing on the Sensitivity Testing and Robustness of the Objective Function¹

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Wildland fire organizations customarily divide the fire suppression problem into stages of management. US federal land managers organize the suppression of unwanted fires into the three stages of initial attack (IA), extended attack (EA) and large fire management. Compartmentalizing the problem allows organizations to focus on the functioning and funding of different levels of fire management.

While compartmentalizing the problem into IA and EA (assume just two for simplicity) provides necessary managerial clarity for planning, budgeting and operations, it can pose a classic externality problem. The correct approach maximizes the sum of the net benefits across both program components (IA and EA). If we could simultaneously both in their entirety, we would know the cost of EA and it could be included in the IA analysis. Only in this way, can we solve for the optimal number of escaped fires. The problem is that the current FPA model was designed to improve seasonal preparedness resource allocation and budgeting for only the initial response (IR) portion of Preparedness, leaving the analysis of EA for a later development phase.

This is identical to the classic pollution problem of a firm producing widgets while polluting the water. If the polluting firm is allowed to use the water at no charge or penalty, then we would expect the firm to over produce widgets and to dump excessive pollution into the stream. The overall cost of producing widgets would be excessive when the cost of pollution is considered. This is known as an externality in the literature and remedies are well known. The principle used to evaluate such remedies was provided in a classic article by Ronald Coase and by the famous “Coase Theorem.” The principle is understood by considering the incentives provided by joint ownership of the two resources. Owing both the factory and the water internalizes the cost of the effluent in the production of widgets. This application of the Coase Theorem is equivalent to maximizing the overall net benefits produced by the factory and the water. This solution is not directly available in the current FPA-PM model because the scope is limited to IA. Specifically, we do not have the benefit and cost information available from modeling the EA problem for use in the IA calculations. Other well-known solutions involve applying penalties or standards on the polluting agent as a proxy for the “Coasian” solution.

¹ Prepared by Rideout, Kirsch and Wei.

We begin with four principles that directly apply to IA modeling:

1. Basic economic theory of cost minimization dictates that there is an optimal number of fires that should escape, just as there is typically an optimal amount of pollution for the polluting firm. A direct corollary is that higher IA success rates do not always minimize total cost, even when the cost of EA is included in the analysis.
2. The optimal number of escapes is unknown. We do not have knowledge of the EA benefits and costs—it was beyond the scope of Phase I.
3. There will always be IA escapes because it is too costly and inefficient to reach 100% containment.
4. IA analysis should have a penalty to reasonably compensate for the cost of suppressing EA fires and for the physical damage of EA fires.

If the IA analysis is addressed in isolation of the EA problem, and the costs (physical damage and suppression costs) of fires that escape IA to become EA events are unrecognized, then we have a bad approach and an inappropriate solution. Because FPA-PM was forced to address the IA problem in isolation, the PM model was developed with a proxy to remedy externality of escaped fires. The objective function was also developed as a strategic level expression of the protection of value at risk across a broad landscape. In the FPA-PM model, these values are at risk from a hypothetical array of fires based on fire history on the landscape. In this context, fires exist solely as a vehicle to address the broader strategic seasonal analysis. There was no expectation that the model would be used to address the management and containment, of individual events.

The management and science reviews of FPA-PM suggest sensitivity testing of alternative objective functions with special regard to initial attack success rate. These reviews questioned the integrity of the objective function; especially regarding initial attack success. These reviews raise potentially serious issues and concerns. The reviews also suggested changing the objective function, but they did not consider the current penalty programmed into FPA-PM to remedy the escaped fire externality.

As suggested, we tested the current FPA-PM objective function against some alternative objective functions, each designed to penalize the IA objective function for escaped fires. Therefore, this paper addresses the penalty used in FPA-PM for escaped fires and alternative ways of introducing this penalty and its implications for interpreting the review process. The results of our testing² are summarized with a discussion and a clear set of conclusions.

² A fully developed paper including test conditions, data and charts is in preparation. Our expected completion date is early summer 2006.

Sensitivity Testing Philosophy and Objective Functions

We tested the following four objective functions (Appendix A) while ignoring fire use as a matter of simplification because our focus here is on containment of unwanted fires.

1. **Current FPA-PM objective function “(18+1)”**

Minimize weighted acres burned and penalize escaped fires by adding “one” weighted acre to the weighted fire size at the time of escape.

2. **Add a large constant penalty “M” to each escaped fire.**

A large acre penalty is added to the objective function and applied every time a fire escapes. The same penalty is applied to every escaped fire. A large or “Big M” is equivalent to maximizing IA success rate.

3. **Add a penalty based on the weighted acres burned at the time of escape.** Here we added a **different penalty to each fire** that was proportionate to each fire’s weighted size at escape. Given the IA scope of analysis, this is the best information available to the model regarding the escape. Weighted size reflects the last known information from IA regarding values at risk, the size of the fire and the likely cost of managing fire in an EA setting. This “baseline” of weighed size at the time of escape can be adjusted upward by adjusting the value of the constant “K” from one to a large number to increase the estimated cost of escapes.

4. **We added “Hard Constraint” to contain a specific number of fires.** This is the most straight forward way of modeling a predetermined, or “mandated” or “target” IA success rate within the FPA-PM framework. Physical production limitations may cause this model to be infeasible with high levels of the constraint. This is not a change to the objective function and it implicitly suggests changing the way that containment is calculated or modeled in the FPA-PM framework.

Findings and Discussion

Objective function (3) best approximates the internalization of escape costs with the information available to the model. It therefore provides a suitable proxy for the penalty of escapes. Objective function (3) provides a “Coasian” benchmark for testing alternative objective functions. Three results were obtained from testing (3)³.

1. The value of $K \geq 1$ has no substantive effect on the number of fires that escape and
2. the value of $K \geq 1$ has no substantive effect on the mix of fires that escape and
3. because a very high penalty (large K) is at work for all values of K, the penalty imposed on escapes is as aggressive as can reasonably be achieved consistent with addressing the protection of values at risk.

Insensitivity of the results with respect to the value of K in this test example can be explained by the following example. When FPA-PM is run, it results in a group of fires that have been contained

³ In the rare case where values of $K > 1$ affect the containment decision, it is likely undesirable to allow K to take on such a value. Where the containment decision is affected by large values of K, the decision will likely be distorted because the objective function will excessively consider the weighted acre burned at escape and thereby stifle the consideration of fire effects during the containment period. In this event, it is best to set the value of K to one.

and a group of fire that have not. First, suppose that two groups of fires are assessed and the model only has enough funds to contain one group. One group will be contained and the other will escape. Next, if the difference in WAB between two groups of fires at escape has already been the determining factor for containment decisions, increasing the value of K will change the absolute value for containment importance for both groups, but it will not change the relative importance between the two groups. There is no resulting change in containment.

Comparing Objective function (1) with objective function (3) provides another crucial finding. ***Objective function (1), as used by FPA-PM (Appendix A), produces effectively⁴ the same effect on escaped fires as objective function (3). Therefore, the current FPA-PM objective function best remedies the potential externality that might be imposed by the cost of escapes.*** Further, because (1) is effectively equivalent to (3) [(1) closely approximates (3) when $K=1$] we directly find that the FPA-PM objective function is very aggressive with respect to containment. In fact, the objective function cannot be more aggressive without losing important information regarding values at risk. The FPA-PM objective function provides a simple way to appropriately and aggressively account for the cost of escaped fires.

Test results also provide an important finding regarding Objective function (2). ***Objective function (2) which administers the same penalty for each escaped fire, is equivalent to maximizing IA success rate when the value of M is large.*** In effect, maximizing IA success rate destroys the information provided to the model on values at risk through the weighting system. Large M, or maximizing containment, is equivalent to making all fires of equal importance to contain when common knowledge is otherwise. Both the theory and the results show that this is an undesirable and potentially costly objective function because it can allow for and encourage important fires to escape while containing relatively unimportant fires (see also Appendix B). Maximizing IA success provides a strong incentive to contain the wrong fires, because it will focus on the cheap and easy ones and they are not always important. These findings will hold so long as there is scarcity in the model, meaning that the cost constraint is sufficiently binding. In the event that the cost constraint is not binding, suggesting that there is no scarcity of resources, then (1) will produce the same results on containment as (2). Therefore, any fires found to escape at very high budget levels are directly attributed to something else in the system such as the modeling of containment effort. This is straightforward test to perform.

Objective function (4), which enables a “hard wire” of the number of fires contained, has the advantage of simplicity, but it does not address the root issue. Use of (4), provides a mechanism to force containment in lieu of addressing containment issues. The dangers of using (4) are: 1) generation of a high rate of infeasible solutions especially for rates suggested as “de facto policy” and 2) disabling sensitivity to the protection of values at risk. (see Appendix B for an example)

The reviews suggest that the objective function may have policy implications regarding how escaped fires are treated. Important features were introduced to the FPA-PM model to address the current policy as expressed in the 1995, 2001, and 2003 federal interagency documents. The FPA-PM objective function directly introduces protection of values at risk across a broader spectrum of values that was not previously available in initial attack modeling. It also introduces a feature for

⁴ While it is mathematically possible to produce a difference, our test results did not produce one. Our results were identical for objective functions (1) and (3).

the benefit of wildland fire use. Reflecting the protection of values at risk in the objective function directly reflects the new policy documents, including the 2003 implementation policy document.⁵ Introducing wildland fire use represents a significant movement in federal IA modeling toward Appropriate Management Response. The FPA-PM objective function reflects current policy by aggressive containment and by directly reflecting the protection of values at risk; including ecosystem values. These important advances are inconsistent with maximizing initial attack success as in objective function (2) which would make the objective function blind to variations in values at risk.

CONCLUSIONS

Sensitivity testing of the FPA-PM objective function suggests three important conclusions:

1. The FPA-PM objective function directly incorporates protection of values at risk in addition to aggressively and appropriately penalizing for escaped fires. In this way, the FPA-PM objective function reflects the federal interagency policy documents of 1995, 2001 and 2003 in ways that maximizing IA success rate could not.
2. The FPA-PM objective function appropriately and aggressively penalizes escaped fires and it is not the source of “excessive” escapes.
3. Issues of IA success are best addressed through analysis of the containment computations. Changing the objective function to increase the number of escapes without addressing containment mechanisms misses the point. Changing the objective function will risk producing a different and potentially costly mix of fires to contain because the objective function already includes the appropriate incentives to contain the correct set of fires.

Evaluation of the sensitivity testing results of alternative objective functions and current policy provide rationale to strongly support the current FPA-PM objective function regarding the mix of fires that might escape IA efforts in the model. Test results confirm that the FPA-PM objective function is appropriately aggressive with respect to containment.

If the model produces 18-hr containment rates that are “too low,”⁶ even at the highest possible cost limits, then the containment effort (fireline production by resources and their interaction with fire perimeter) or the definition of IA success should be assessed; not the penalty for escapes currently in the objective function.

⁵ From the 2003 “Interagency Strategy for the Implementation of Federal Wildland Fire Management Policy (p23), the definition of initial attack is: “Initial Attack – An aggressive suppression action consistent with firefighter and public safety and values to be protected.”

⁶ Too low is subjective because there are currently no data on fires contained in 18 hours to support such a claim.

Appendix A: Objective functions tested

1. Penalize each escaped fire by adding 1 acre to the weighted area burned at the end of initial attack period. The OF is:

$$\text{Minimize} \quad \sum_{i=1}^I \sum_{d=1}^D (W_{id} \times f_{id} \times A_{id}) + \sum_{i=1}^I (W_{iD} \times f_{iD_e} \times (A_{iD} + 1))$$

This reflects the IA portion of the OF used in the current FPA. The additional 1 acre makes sure that there always some additional benefit of containing a fire if the budget is available. The information of relative importance of fires before escaping will be maintained and therefore important fires will likely be contained.

2. Penalize each escaped fire by using a large constant penalty “M”. The OF is:

$$\text{Minimize} \quad \sum_{i=1}^I \sum_{d=1}^D (W_{id} \times f_{id} \times A_{id}) + M \times \sum_{i=1}^I f_{iD_e}$$

Where terms are defined as usual except that M is the per escaped fire penalty. As M becomes very large, this OF effectively becomes maximizing initial attack success rate where important fires may not be contained. It treats all escaped fires as the same by penalizing them all with the same M (regardless of the weight or size).

3. Penalize each escaped fire by assuming a linear increase of its weighted size at the time of escape. This objective function is:

$$\text{Minimize} \quad \sum_{i=1}^I \sum_{d=1}^D (W_{id} \times f_{id} \times A_{id}) + K \times \sum_{i=1}^I (W_{iD} \times f_{iD_e} \times A_{iD})$$

Fires with higher weighted acres burned at escape would be more important to be aggressively managed during the IA period. K is a constant that can be varied from at least one to a large number.

4. Adding a hard constraint to O.F. 3 to restrict that the number of fires (or percentage) to escape cannot be more than N. Then the objective function and additional constraint will be:

$$\text{Minimize} \quad \sum_{i=1}^I \sum_{d=1}^D (W_{id} \times f_{id} \times A_{id}) + K \times \sum_{i=1}^I (W_{iD} \times f_{iD_e} \times A_{iD})$$

St :

$$\sum_{i=1}^I f_{iD_e} \leq N$$

“N” physically restricts the number of fires that would escape.

Appendix B: Test Results

We tested the four objective functions on a fictitious fire scenario and extensive sensitivity testing of the modeled parameters. The modeled results are below with a discussion of the findings. For this paper refer to the following example in the discussions:

Fire	WAB @ escape
A	5
B	10
C	11
D	19
E	17

Table 1

1. **Objective function three**, provides an economically consistent benchmark: the value of the constant ‘K’ does not affect the number of escapes and it does not affect which fires escape if the difference of WAB between fires at their escape has already been the determining factor.

The reason for this is difficult to understand, but it is because the proportionate penalty of escapes is unchanged in the analysis. Because K is the same for all fires, changing the value of K won’t influence the relative importance of the escaped fires.

We use a five-fire example for demonstration (Table 1). If any fire A, B, C, D and E is contained, we assume the WAB of that fire is zero. If any of the five fires escaped, the WAB for each escape is shown in table 1. Suppose at a given budget level, either three smaller fires A, B, C can be contained, or two larger fires D, E can be contained, then no matter what the value of K is, the model will try to contain fires D and E because this gives a total WAB of $K*26$. It will not contain fires A, B and C because it will create a total WAB of $K*36$, which is always greater than $K*26$.

The conclusion that K does not affect the containment decisions might not hold under rare circumstances. For example, changing the above example by assuming that if any fire A, B, and C is contained, the WAB of each fire is zero; if any fire D and E is contained, the WAB of each fire is 10. This could represent the case that there are much longer dispatch distances to both D and E. In this example, if $K=1$, fire A, B and C will be contained since the WAB is $0+36 = 36$, which is smaller than containing D and E with a WAB of $20+26 = 46$. However, if $K=3$, fires A, B and C will not be contained because the total WAB is $0+3*36 = 108$, which is larger than the WAB of $20+3*26 = 98$ by containing D and E. However, in this case, using a large K, i.e. 3, might not be desirable since it causes more actual WAB during the initial attack period. In addition, it would cause a lower initial attack success rate.

2. **Objective function one** is effectively identical to objective function three for K equal to one. Remember, in most cases, the value of K is irrelevant to containment.

By using the same five-fire example in table 1, the model will always contain fire D and E because this will give a total WAB of $29 = 26+3$. It will not contain A, B and C because it will create a total WAB of $38 = 36+2$.

3. **Objective function two** (big M only) treats all fires the same at escape and is effectively the same as maximizing initial attack success rate assuming M is very large. If M is large enough, the weights become negligible, letting important fires escape to increase the success rate.

For example, if M=1000 and the budget level allows us to contain either fires A,B,C or D and E, the model will choose to contain A,B, and C for a WAB of 2036. The model will not choose to contain the important fires (D, and E) because the WAB will be 3026. This is apparently not what we want.

4. **Objective function (constraint) four** forces the model to either contain the specified number of fires or to go infeasible. Hard-wiring the success rate means sacrificing the containment of important fires.

By using the same 5 fires example, if upper bound for the number of escaped fire is set to 3, the model will contain D and E and the total WAB is 26. Decreasing the number of escaped fires to 2 the model will contain A, B and C, and the total WAB is 36. Here it shows that a higher success rate would create containments with a higher WAB (lower is better).

Appendix C: Why IA Success in FPA-PM Should Differ from Practice

There are three principle reasons why IA success rate should not be expected to approximate success rates in practice. They are:

1. While the model was constructed to be consistently cost effective, real life fire management is not. The job of tactically managing unwanted fires is to initially achieve containment. This may produce IA success rates higher than those in the strategic model that is focused on seasonal performance and budgeting, not individual fire management.
2. The model used a “hard” budget constraint. Unit managers’ work during the season with a “semi” unconstrained budget. Well known tools of severity funding, and the involvement of a “militia of non-fire funded personnel who are trained to fight fire as collateral duty, and co-operators will provide a higher IA success rate than should be expected in a cost-constrained model.
3. The metric of success used by many agencies is more liberal than the FPA metric which is based upon a strict 18 hour period. Agencies differ in their criteria for IA success where it is common to see 48 hour periods or even acreage definitions.

While additional factors that make the comparison of IA success in FPA-PM incongruent with practice, these three likely account for the greatest expected differences. While the extent of each is unknown, it is not unreasonable to suggest that each one might account for about a 10% difference. If the current IA success rate is 95%, then a reasonable expectation of comparison for the FPA-PM analysis would be in the neighborhood of 65%. The implication of this is not that we should expect a well functioning model to attain 65% IA success, but that it is unreasonable to expect that a well functioning PM model should achieve a success rate in the neighborhood of 95-100 percent. It is unproductive to hold FPA-PM to an unreasonable standard of 95-99% IA success. Instead, translating success in the model with that observed in practice will improve understanding of the model and better enable those interested in containment results to focus on the containment calculation.